Kako je binomni obrazac vrlo dobro poznat:

$$\frac{\mathrm{d}^k}{\mathrm{d}x^k}(uv) \equiv \binom{k}{0}\frac{\mathrm{d}^k u}{\mathrm{d}x^k}\frac{\mathrm{d}^0 v}{\mathrm{d}x^0} + \binom{k}{1}\frac{\mathrm{d}^{k-1} u}{\mathrm{d}x^{k-1}}\frac{\mathrm{d}^1 v}{\mathrm{d}x^1} + \binom{k}{2}\frac{\mathrm{d}^{k-2} u}{\mathrm{d}x^{k-2}}\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} + \ldots + \binom{k}{k-2}\frac{\mathrm{d}^2 u}{\mathrm{d}x^2}\frac{\mathrm{d}^{k-2} v}{\mathrm{d}x^{k-2}} + \binom{k}{k-1}\frac{\mathrm{d}^1 u}{\mathrm{d}x^1}\frac{\mathrm{d}^{k-1} v}{\mathrm{d}x^{k-1}} + \binom{k}{k}\frac{\mathrm{d}^0 u}{\mathrm{d}x^0}\frac{\mathrm{d}^k v}{\mathrm{d}x^k}$$

(naravno, nulti izvod funkcije je sama ta funkcija, $\frac{\mathrm{d}^0 v}{\mathrm{d}x^0} \equiv v$) , pa je npr.

$$\frac{\mathrm{d}^{k+1}}{\mathrm{d}x^{k+1}}\left((1-x^2)y'\right) \equiv \binom{k+1}{k-1}\frac{\mathrm{d}^2(1-x^2)}{\mathrm{d}x^2}\frac{\mathrm{d}^{k-1}y'}{\mathrm{d}x^{k-1}} + \binom{k+1}{k}\frac{\mathrm{d}^1(1-x^2)}{\mathrm{d}x^1}\frac{\mathrm{d}^k y'}{\mathrm{d}x^k} + \binom{k+1}{k+1}\frac{\mathrm{d}^0(1-x^2)}{\mathrm{d}x^0}\frac{\mathrm{d}^{k+1}y'}{\mathrm{d}x^{k+1}},$$

(svi ostali clanovi su uginuli, jer sadrze treci ili vise izvode od $1-x^2$ sto je nula) odnosno:

$$\frac{\mathrm{d}^{k+1}}{\mathrm{d}x^{k+1}}\left((1-x^2)y'\right) \equiv -zk(k+1) - 2xz'(k+1) + (1-x^2)z''.$$

Slicno se dobija i da je $\frac{\mathrm{d}^{k+1}}{\mathrm{d}x^{k+1}}(2lxy) = 2l\left((k+1)z + xz'\right)$, pa se sabiranjem dobija 9.1.44.

Frobeniusova metoda.

$$y'' + y = 0$$

$$y = \sum_{i=0}^{\infty} a_i x^i = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 \ldots = \sum a_i x^i = \sum_{i=0} a_i x^i$$

$$y' = 1 \cdot a_1 x^0 + 2a_2 x^1 + 3a_3 x^2 + 4a_4 x^3 + \ldots$$
$$y'' = 2a_2 x^0 + 6a_3 x^1 + 12a_4 x^2 + \ldots$$

$$y' = \sum_{i=1} i a_i x^{i-1},\ y'' = \sum_{i=2} i(i-1)a_i x^{i-2}$$

$$\sum_{i=0} a_i x^i + \sum_{i=2} i(i-1)a_i x^{i-2} = a_0 + a_1 x + \sum_{i=2}\left(a_i x^i + i(i-1)a_i x^{i-2}\right) = \sum_{i=0}\left(a_i x^i + i(i-1)a_i x^{i-2}\right) = 0$$

	Isuma	*IIsuma*
x^0	$i=0; a_0$	$i=2; 2a_2$
x^1	$i=1; a_1$	$i=3; 6a_3$
x^2	$i=2; a_2$	$i=4; 12a_4$
x^3	$i=3; a_3$	$i=5; 20a_5$
x^4	$i=4; a_4$	$i=6; 30a_6$
$\vdots$	$\vdots$	$\vdots$

$\rightarrow$

$a_0 + 2a_2 = 0$
$a_1 + 6a_3 = 0$
$a_2 + 12a_4 = 0$
$a_3 + 20a_5 = 0$
$a_4 + 30a_6 = 0$
$\vdots$

$\rightarrow$

$a_2 = -a_0 / 2$
$a_3 = -a_1 / 6$
$a_4 = -a_2 / 12$
$a_5 = -a_3 / 20$
$a_6 = -a_4 / 30$
$\vdots$

$$\rightarrow a_{k+2} = -\frac{a_k}{(k+2)(k+1)}$$

$$\sum_{i=0} a_i x^i + \sum_{i=2} i(i-1) a_i x^{i-2} = 0$$

$$\downarrow$$

$$i = k \quad ; \quad i = k+2$$

$$a_k x^k + (k+2)(k+1) a_{k+2} x^k = 0$$

$$\downarrow$$

$$a_{k+2} = -\frac{a_k}{(k+2)(k+1)}$$

$$\left(a_0, -\frac{a_0}{2}, +\frac{a_0}{24}, -\frac{a_0}{720}, \dots \right) = \left(a_0, -\frac{a_0}{2!}, +\frac{a_0}{4!}, -\frac{a_0}{6!}, \dots \right)$$

$$\left(a_1, -\frac{a_1}{6}, +\frac{a_1}{120}, -\frac{a_1}{5040}, \dots \right) = \left(a_1, -\frac{a_1}{3!}, +\frac{a_1}{5!}, -\frac{a_1}{7!}, \dots \right)$$

$$y = \sum_{i=0}^{\infty} a_i x^i = a_0 x^0 + a_1 x^1 - \frac{a_0}{2!} x^2 - \frac{a_1}{3!} x^3 + \frac{a_0}{4!} x^4 + \frac{a_1}{5!} x^5 - \frac{a_0}{6!} x^6 - \frac{a_1}{7!} x^7 + \dots \left(a_0, a_1 \in \mathbb{C} \right)$$

$$y = y_0 + y_1 = a_0 \cos x + a_1 \sin x \quad \left(a_0, a_1 \in \mathbb{C} \right)$$