Domaći zadatak 3, Uvod u laboratorijski rad

1. Izvesti izraz za grešku ako je *y* zavisno promenljiva, dok su *x*, *z* i *p* nezavisno promenljive:

a) $y = 18x + 16mp$

b) $y = \frac{8,5 \cdot x \cdot z^2}{p}$

c) $y = \sin(x) + 18\frac{m^2}{p}$

d) $y = e^{4m} + \ln(8p)$

2. Izračunati koncentraciju rastvora i predstaviti izraz sa nesigurnošću ako je odmereno (0,199 ± 0,001) g kalijum-nitrata i rastvoreno u sudu zapremine (100 ± 0,05) mL. (A(K) = 39,0983 $gmol^{-1}$, A(N) = 14,0067 $gmol^{-1}$ i A(O) = 15,999 $gmol^{-1}$)

3. Koji od dva ponuđena postupka razblaženja osnovnog $c_{početno}$ = (1,00 ± 0,01) M rastvora do $c_{krajnje}$ = 0,001 M rastvora daje manju ukupnu neodređenost:

a) Prenošenje pipetom od $V_{početno}$ = (1,000 ± 0,006) mL i razblaženje u normalnom sudu od $V_{krajnje}$ = (1000,00 ± 0,30)mL

b) Prenošenje pipetom od $V_{početno}$ = (20,00 ± 0,03) mL i razblaženje u normalnom sudu od (100,00 ± 0,08) mL i nakon toga prenošenje pipetom od (1,000 ± 0,006) mL i razblaženje u normalnom sudu do $V_{krajnje}$ = (500,00 ± 0,20) mL.

Jednačina razblaženja je:

$$c_{krajnje} = \frac{c_{početno} \cdot V_{početno}}{V_{krajnje}}$$

4. Odrediti broj molova idealnog gasa pri sledećim uslovima: P=(5,92 ± 0,05) atm; V=(15,0 ± 0,1) L; T=(25,0 ± 0,5) ºC i R=8,314 J/KmoL. Rezultat prikazati sa nesigurnošću.

5. Izračunati zapreminu koju zauzima (120,06 ± 0,06) g ugljen-dioksida, pod pretpostavkom da se ovaj gas ponaša prema jednačinama idealnog gasnog stanja, i rezultat prikazati sa nesigurnošću. (A(C)=12,01 g/mol i A(O)=16,00 g/mol). Koristiti formulu:

$$V = V_m \cdot \frac{m}{M}$$

V_m je zapremina jednog mola idealnog gasa i iznosi 22,4 dm^3.

6. Prikazati rezultat prema pravilima o prikazivanju rezultata i zaokruživanju grešaka ako su prilikom merenja dobijene sledeće relativne greške:

a) jačina struje je 45,86 A sa relativnom greškom od 35 %
b) masa iznosi 1236 g sa relativnom greškom od 0,08 %

Домаћи 3
решења са поступком.

1. x, z и P → независно променљиве
ω → константа
y → зависно променљива

a) $y = 18x + 16\omega P$

$$dy = \left(\frac{\partial y}{\partial x}\right)_P \cdot dx + \left(\frac{\partial y}{\partial P}\right)_x \cdot dP$$

$$\left(\frac{\partial y}{\partial x}\right)_P = \frac{\partial(18x + 16\omega P)}{\partial x} = 18$$

$$\left(\frac{\partial y}{\partial P}\right)_x = \frac{\partial(18x + 16\omega P)}{\partial P} = 16\omega$$

$$dy = 18 \cdot dx + 16\omega \cdot dP$$

$$\Delta y = 18\Delta x + 16\omega \Delta P$$

b) $y = \dfrac{8{,}5 \cdot x \cdot z^2}{P} \quad / \ln$

$$\ln y = \ln 8{,}5 + \ln x + 2\ln z - \ln P \quad / \text{диференцирамо}$$

$$(\ln y)' dy = (\ln 8{,}5)' d8{,}5 + (\ln x)' dx + (2\ln z)' dz - (\ln P)' dP$$

$$\frac{dy}{y} = \frac{dx}{x} + 2\frac{dz}{z} + \frac{dP}{P}$$

$$\Delta y = y \cdot \left(\frac{\Delta x}{x} + 2\frac{\Delta z}{z} + \frac{\Delta P}{P}\right)$$

c) $y = \sin(x) + 18\frac{u2}{p}$

$$dy = \left(\frac{\partial y}{\partial x}\right)_p dx + \left(\frac{\partial y}{\partial p}\right)_x dp$$

$$\left(\frac{\partial y}{\partial x}\right)_p = \left(\frac{\partial(\sin x + \frac{18u^2}{p})}{\partial x}\right) = \cos x$$

$$\left(\frac{\partial y}{\partial p}\right)_x = \frac{\partial\left(\sin x + \frac{18u2}{p}\right)}{\partial p} \cdot = -\frac{18u2}{p2}$$

$$dy = \cos x\, dx + \frac{-18u2}{p2} \cdot dp$$

$$\Delta y = |\cos x| \Delta x + \left|\frac{-18u2}{p2}\right| \Delta p$$

$$\Delta y = |\cos x| \Delta x + \frac{18u2}{p2} \Delta p$$

d) $y = e^{4u} + \ln(8p)$

$$dy = \left(\frac{\partial y}{\partial p}\right) \cdot dp = \frac{\partial(\ln(8p))}{\partial p} dp = \left(\ln(8p)\right)' \cdot (8p)' \cdot dp$$

$$du_c = \frac{8}{8p} dp$$

$$\Delta y = \frac{\Delta p}{p}$$

2. $C = ?$

$\Delta C = ?$

$(C \pm \Delta C) = ?$

$m(KNO_3) = (0{,}199 \pm 0{,}001)\,g$

$V_R(KNO_3) = (100 \pm 0{,}05)\,ml$

$A(K) = 39{,}0983\ g/mol$

$A(N) = 14{,}0067\ g/mol$

$A(O) = 15{,}999\ g/mol$

$$C_R(KNO_3) = \frac{n(KNO_3)}{V_R(KNO_3)} = \frac{m(KNO_3)}{M(KNO_3) \cdot V_R(KNO_3)}$$

$$M(KNO_3) = A(K) + A(N) + 3 \cdot A(O) = 39{,}0983\ g/mol + 14{,}0067\ g/mol + 3 \cdot 15{,}999\ g/mol$$

$M(KNO_3) = 101{,}102\ g/mol$

$$C_R(KNO_3) = \frac{0{,}199\ g}{101{,}102\ g/mol \cdot 0{,}1\ l} = 0{,}0197\ M$$

$$c_R(KNO_3) = \frac{m(KNO_3)}{M(KNO_3) \cdot V_R(KNO_3)} \quad / \ln$$

$$\ln C_R(KNO_3) = \ln m(KNO_3) - \ln M(KNO_3) - \ln V_R(KNO_3) \quad / d$$

$$\frac{\Delta C_R(KNO_3)}{C_R(KNO_3)} = \frac{\Delta m(KNO_3)}{m(KNO_3)} + \frac{\Delta M(KNO_3)}{M(KNO_3)} + \frac{\Delta V_R(KNO_3)}{V_R(KNO_3)}$$

$$\Delta M(KNO_3) = \Delta A(K) + \Delta A(N) + 3\Delta A(O) = (0{,}00014 + 0{,}0001 + 3 \cdot 0{,}001)\ g/mol$$

$\Delta M(KNO_3) = 0{,}0032\ g/mol$

$$\Delta C_R(KNO_3) = 0{,}0197\ M \cdot \left(\frac{0{,}001}{0{,}199} + \frac{0{,}0032}{101{,}102} + \frac{0{,}05}{100} \right)$$

$$\Delta C_R(KNO_3) = 0{,}0197\ M \cdot (0{,}005025 + 0{,}000317 + 0{,}0005)$$

2. $\Delta C(KNO_3) = 0{,}01197\,M \cdot 0{,}005842 = 0{,}00011308$

$C_R(KNO_3) = (0{,}0197 \pm 0{,}0002)\,M$

3.

$C_{početno} = (1{,}00 \pm 0{,}01)\,M$

$C_{KRAJNJE} = 0{,}001\,M$

a) $V_{početno} = (1{,}000 \pm 0{,}006)\,l$

$V_{KRAJNJE} = (1000 \pm 0{,}130)\,ml$

$C_{poč} \cdot V_{poč} = C_{KRAJ} \cdot V_{KRAJ}$

$C_{KRAJ} = \frac{C_{poč} \cdot V_{poč}}{V_{KRAJ}} \quad / \ln$

$\ln C_{KRAJ} = \ln C_{poč} + \ln V_{poč} - \ln V_{KRAJ} \quad / d$

$\frac{\Delta C_{KRAJ}}{C_{KRAJ}} = \frac{\Delta C_{poč}}{C_{poč}} + \frac{\Delta V_{poč}}{V_{poč}} + \frac{\Delta V_{KRAJ}}{V_{KRAJ}}$

$\Delta C_{KRAJ} = C_{KRAJ} \cdot \left(\frac{\Delta C_{poč}}{C_{poč}} + \frac{\Delta V_{poč}}{V_{poč}} + \frac{\Delta V_{KRAJ}}{V_{KRAJ}} \right)$

$\Delta C_{KRAJ} = 0{,}001\,M \cdot \left(\frac{0{,}01}{1{,}00} + \frac{0{,}006}{1{,}000} + \frac{0{,}130}{1000} \right)$

$\Delta C_{KRAJ} = 0{,}001\,M \cdot (0{,}01 + 0{,}006 + 0{,}0003) = 0{,}001\,M \cdot 0{,}0163$

$\Delta C_{KRAJ} = 1{,}63 \cdot 10^{-5}\,M$

$C_{KRAJ} = (1{,}00 \pm 0{,}02) \cdot 10^{-3}\,M$

b) I КОРАК

СТАНДАРДНИ РАСТВОР

$V^{I}_{поч} = (20{,}00 \pm 0{,}03)\,mL$

$V^{I}_{крај} = (100{,}00 \pm 0{,}08)\,mL$

$C^{I}_{поч} = (1{,}00 \pm 0{,}01)\,M$

$$C^{I}_{крај} = \frac{C^{I}_{поч} \cdot V^{I}_{поч}}{V^{I}_{крај}} = \frac{1{,}00\,M \cdot 20{,}00\,mL}{100{,}00\,mL} = 0{,}2\,M$$

$$\Delta C^{I}_{крај} = \left(\frac{\Delta C^{I}_{поч}}{C^{I}_{поч}} + \frac{\Delta V^{I}_{поч}}{V^{I}_{поч}} + \frac{\Delta V^{I}_{крај}}{V^{I}_{крај}} \right) \cdot C^{I}_{крај}$$

$$\Delta C^{I}_{крај} = \left(\frac{0{,}01}{1{,}00} + \frac{0{,}03}{20{,}00} + \frac{0{,}08}{100{,}00} \right) = (0{,}01 + 0{,}0015 + 0{,}008)\,0{,}2\,M$$

$$\Delta C^{I}_{крај} = 0{,}0123 \cdot 0{,}2\,M = 0{,}00246\,M$$

II КОРАК

$C^{II}_{поч} = C^{I}_{крај} = (0{,}200 \pm 0{,}003)\,M$

$V^{II}_{поч} = ?$

$V^{II}_{крај} = (500{,}00 \pm 0{,}20)\,mL$

$C^{II}_{крај} = 0{,}001$

$$V^{II}_{поч} = \frac{C^{II}_{крај} \cdot V^{II}_{крај}}{C^{II}_{поч}} = \frac{0{,}001\,M \cdot 500{,}00\,mL}{0{,}200\,M} = 2{,}5\,mL$$

са пипетом од 1 mL потребно је мерити 3 x за пречу
једном од 0,5 mL а другта 2 пута по 1 mL

То значи за одмеравање 2,5 mL направи се
тростурука грешка $3 \cdot 0{,}006\,mL = 0{,}018\,mL$

$V^{II}_{поч} = (2{,}50 \pm 0{,}02)\,mL$

$$\Delta C^{II}_{крај} = \left(\frac{\Delta C^{II}_{поч}}{C^{II}_{поч}} + \frac{\Delta V^{II}_{поч}}{V^{II}_{поч}} + \frac{\Delta V^{II}_{крај}}{V^{II}_{крај}} \right) \cdot C^{II}_{крај}$$

$$\Delta C_{KPAJ}^{I} = \left(\frac{0{,}003}{0{,}200} + \frac{0{,}02}{2{,}50} + \frac{0{,}20}{500{,}00}\right) \cdot 0{,}001\,M$$

$$\Delta C_{KPAJ}^{II} = (0{,}015 + 0{,}008 + 0{,}0004) \cdot 0{,}001\,M$$

$$\Delta C_{KPAJ}^{I} = 0{,}0234 \cdot 0{,}001\,M = 2{,}34 \cdot 10^{-5}\,M$$

$$C_{KPAJ}^{I} = (1{,}00 \pm 0{,}03) \cdot 10^{-3}\,M$$

ЗАКЛУЧАК II

$\Delta C_{KPAJ} < \Delta C_{KPAJ}$

ПОСТИЧКОМ ПОД а) ПРАЗИ СЕ МАЊА НЕСИГУРНОСТ

4. $p = (5{,}92 \pm 0{,}05)\,atm = (599844 \pm 5066{,}25)\,Pa$

$V = (15{,}0 \pm 0{,}1)\,L = (15{,}0 \pm 0{,}1) \cdot 10^{-3}\,m^3$

$T = (25{,}0 \pm 0{,}5)\,^\circ C = (298{,}16 \pm 0{,}5)\,K$

$R = 8{,}314\,J/kmol$

$pV = nRT$

$$n = \frac{pV}{RT} = \frac{599844\,Pa \cdot 15 \cdot 10^{-3}\,m^3}{8{,}314\,J/kmol \cdot 298{,}16\,K} = 3{,}6297\,mol$$

$n = \frac{pV}{RT} \quad / \ln$

$\ln n = \ln p + \ln V - \ln R - \ln T \quad / d$

$(\ln n)' dn = (\ln p)' dp + (\ln V)' dV - (\ln R)' dR - (\ln T)' dT$

(константа → 0)

$$\frac{\Delta n}{n} = \frac{\Delta p}{p} + \frac{\Delta V}{V} + \frac{\Delta T}{T}$$

$$\Delta n = n\left(\frac{\Delta p}{p} + \frac{\Delta V}{V} + \frac{\Delta T}{T}\right)$$

$$\Delta n = 3{,}6297\,mol \cdot \left(\frac{0{,}05}{5{,}92} + \frac{0{,}1}{15} + \frac{0{,}5}{298{,}16}\right)$$

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4. $\Delta n = 3{,}6297\,mol \cdot (0{,}00845 + 0{,}00667 + 0{,}001677)$

$\Delta n = 3{,}6297\,mol \cdot 0{,}016797 = 0{,}06097\,mol$

$n = (3{,}63 \pm 0{,}07)\,mol$

5.

$m(CO_2) = (120{,}06 \pm 0{,}06)\,g$

$V(CO_2) = ?$

$A(C) = 12{,}01\,g/mol$; $A(O) = 16{,}00\,g/mol$

$M(CO_2) = A(C) + 2A(O) = 12{,}01\,g/mol + 2 \cdot 16{,}00\,g/mol$

$M(CO_2) = 44{,}01\,g/mol$

$\Delta M(CO_2) = \Delta A(C) + 2\Delta A(O) = 0{,}01\,g/mol + 2 \cdot 0{,}01\,g/mol$

$\Delta M(CO_2) = 0{,}03\,g/mol$

$V(CO_2) = V_m \cdot n(CO_2) = V_m \cdot \frac{m(CO_2)}{M(CO_2)} = 22{,}4\,dm^3 \frac{120{,}06\,g}{44{,}01\,g/mol}$

$V(CO_2) = 61{,}108\,dm^3$

$V(CO_2) = V_m \frac{m(CO_2)}{M(CO_2)} \quad / \ln$

$\ln V(CO_2) = \ln V_m + \ln m(CO_2) - \ln M(CO_2) \quad / d$

$(\ln V(CO_2))' d V(CO_2) = (\ln V_m)' dV_m + (\ln m(CO_2))' dm(CO_2) - (\ln M(CO_2))' \cdot dM(CO_2)$

$\frac{\Delta V(CO_2)}{V(CO_2)} = \frac{\Delta m(CO_2)}{m(CO_2)} + \frac{\Delta M(CO_2)}{M(CO_2)}$

5.

$$\Delta V(CO_2) = V(CO_2) \cdot \left(\frac{\Delta m(CO_2)}{m(CO_2)} + \frac{\Delta M(CO_2)}{M(CO_2)} \right)$$

$$\Delta V(CO_2) = 61{,}108\ dm^3 \cdot \left(\frac{10{,}06}{120{,}06} + \frac{0{,}03}{44{,}01} \right)$$

$$\Delta V(CO_2) = 61{,}108\ dm^3 \cdot (4{,}998 \cdot 10^{-4} + 6{,}817 \cdot 10^{-4})$$

$$\Delta V(CO_2) = 61{,}108\ dm^3 \cdot 11{,}815 \cdot 10^{-4} = 721{,}99 \cdot 10^{-4}\ dm^3$$

$$\Delta V(CO_2) = 0{,}0722\ dm^3$$

$$V(CO_2) = (61{,}11 \pm 0{,}08)\ dm^3$$

16.

a) $\underline{I} = 45{,}86\ A$

$\delta \underline{I} = 3{,}5\ \%$

$\Delta I = \underline{I} \cdot \delta \underline{I} = 45{,}86\ A \cdot 0{,}035 = 16{,}051\ A$

b) $m = 1236\ g$

$\delta m = 0{,}08\ \%$

$\Delta m = m \cdot \delta m = 1236\ g \cdot 0{,}0008 = 0{,}9888\ g$

$m = (1236 \pm 1)\ g$