Физика 1
предавање (8.5.2020.)

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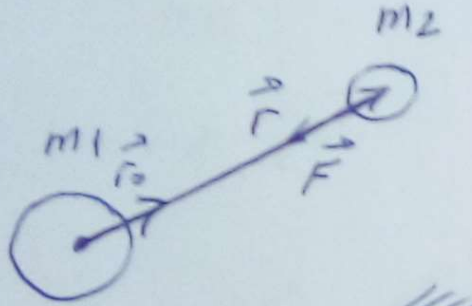
Њутнов закон гравитације

$$\vec{F} = -\gamma \frac{m_1 \cdot m_2}{r^2} \cdot \vec{r}_0 \qquad \gamma = 6{,}67 \cdot 10^{-11} \frac{Nm^2}{kg^2}$$

универзална гравитациона константа



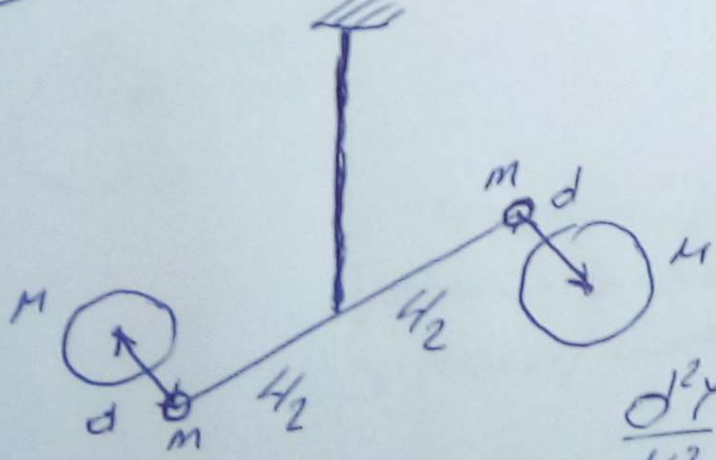
Кевендиш (Cavendish) 1798

Торзиона вага:

момент силе

торзиона константа

$$M = 2 \cdot \frac{L}{2} \cdot \gamma \frac{mM}{d^2} = k\varphi$$



$$J \cdot \frac{d^2\varphi}{dt^2} = -k\varphi \qquad \left(J = 2m\left(\frac{L}{2}\right)^2 \right)$$

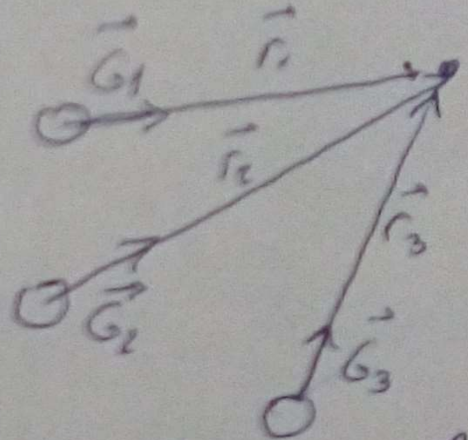
$$\frac{d^2\varphi}{dt^2} + \underbrace{\left(\frac{k}{J}\right)}_{\omega^2}\varphi = 0 \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{J}{k}} = 2\pi\sqrt{\frac{mL^2}{2k}}.$$

Јачина гравитационог поља

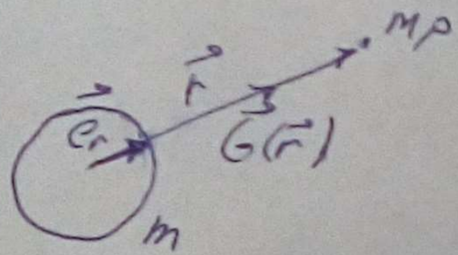
$$\vec{F} = m_p \cdot \vec{G}$$

Закон суперпозиције!

$$\Rightarrow \quad \vec{G} = \frac{\vec{F}}{m_p} = -\gamma \frac{\cancel{m_p} m}{r^2} \vec{e}_r \cdot \frac{1}{\cancel{m_p}}$$



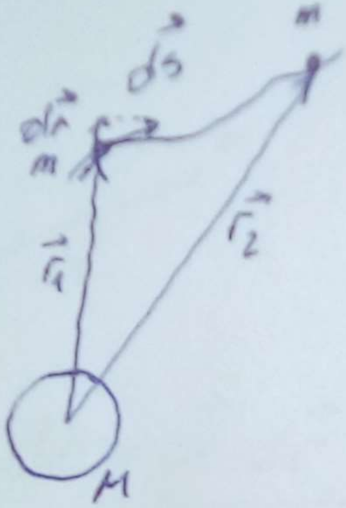
$$\vec{G} = -\gamma \frac{m}{r^2} \vec{e}_r$$



$$\vec{G} = \sum_{i=1}^{n} \vec{G}_i = \sum_{i=1}^{n} \frac{\vec{F}_i}{m_p} = -\sum_{i=1}^{n} \gamma \frac{m_i}{r_i^2} \vec{e}_{r_i}$$

укупна јачина гравитационог поља од свих тела у некој тачки простора.

Рад гравитационе силе



$$dA = \vec{F} \cdot d\vec{s} = -\gamma \frac{mM}{r^2} \underbrace{\vec{e}_r \cdot d\vec{s}}_{dr}$$

$$A_{12} = \int_{r_1}^{r_2} -\gamma \frac{mM}{r^2} dr = \gamma mM \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$dA = -dU \Rightarrow A_{12} = U_1 - U_2 = -\gamma mM \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

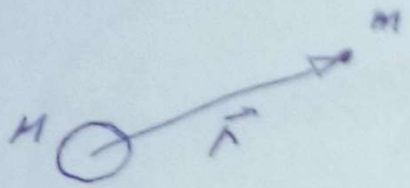
$$r_2 \to \infty \Rightarrow U = U_1 = -\gamma mM \frac{1}{r_1}$$

потенцијална енергија тела масе m у гравитационом пољу тела масе M.

означава да је тело масе m заробљено у гравитационом пољу тела масе M.

Потенцијална енергија и гравитациони потенцијал, градијент потенцијала

$$U(\vec{r}) = -\gamma \frac{M \cdot m}{r} \quad \text{– потенцијална енергија}$$



$$\varphi(\vec{r}) = \frac{U(\vec{r})}{m} = -\gamma \frac{M}{r} \quad \text{– гравитациони потенцијал}$$

$$\underbrace{dA = \vec{F} \cdot d\vec{r} = -dU}_{} \Rightarrow \vec{F} = -\frac{dU}{d\vec{r}} = -\underline{grad}\, U = -\left(\frac{\partial U}{\partial x}\vec{e}_x + \frac{\partial U}{\partial y}\vec{e}_y + \frac{\partial U}{\partial z}\vec{e}_z\right)$$

Како је:

$$\varphi = \frac{U}{m} \quad \text{и} \quad \vec{G} = \frac{\vec{F}}{m}$$

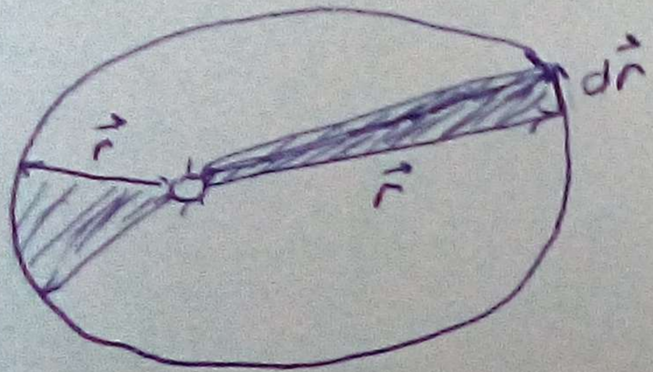
$$\underline{grad} = \left(\frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}e_z\right)$$

оператор градијента

Слично:

$$\vec{G} = -\underline{grad}\,\varphi = -\left(\frac{\partial \varphi}{\partial x}\vec{e}_x + \frac{\partial \varphi}{\partial y}\vec{e}_y + \frac{\partial \varphi}{\partial z}\vec{e}_z\right)$$

Кеплерови закони



1° Елиптичне путање планета

2° $$\frac{d\vec S}{dt} = \frac{1}{2}\frac{\vec r \times d\vec r}{dt} = \frac{1}{2m}\left(\vec r \times m\frac{d\vec r}{dt}\right) = \frac{\vec L}{2m} = \overrightarrow{const}$$

$$\frac{d\vec L}{dt} = \vec M \; ; \quad \vec M = 0 \Rightarrow \frac{d\vec L}{dt} = 0$$

$$\Rightarrow \vec L = \overrightarrow{const}.$$

3° $a^2 \sim T^3$

Кеплерови закони

$$E = E_k + U(r)$$

$$E = \frac{m}{2}\Big[\underbrace{\left(\frac{dr}{dt}\right)^2}_{v_r^2} + \left(\frac{L}{mr}\right)^2\Big] + U(r)$$

$$\left(\frac{mr^2\frac{d\theta}{dt}}{mr}\right)^2 = \left(r\frac{d\theta}{dt}\right)^2 = (v_\tau)^2$$

$$\Rightarrow \quad \frac{dr}{dt} = \sqrt{\frac{2(E-U(r))}{m} - \left(\frac{L}{mr}\right)^2}$$

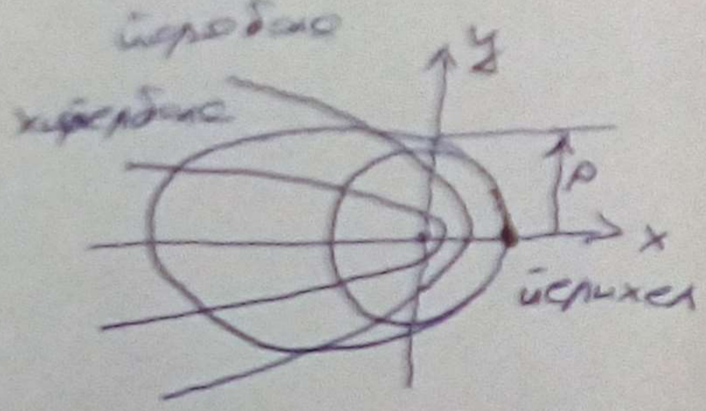
$$dt = \frac{dr}{\sqrt{2(E-U(r))/m - \left(\frac{L}{mr}\right)^2}}, \quad \frac{d\theta}{dt} = \frac{L}{mr^2}$$

$$\Rightarrow \quad \boxed{d\theta = \frac{L\cdot dr}{r^2\cdot\sqrt{2m(E-U(r)) - \left(\frac{L}{r}\right)^2}}}$$

$$r = \frac{p}{1 - e\cdot\cos\theta}$$

p - фокусен параметър

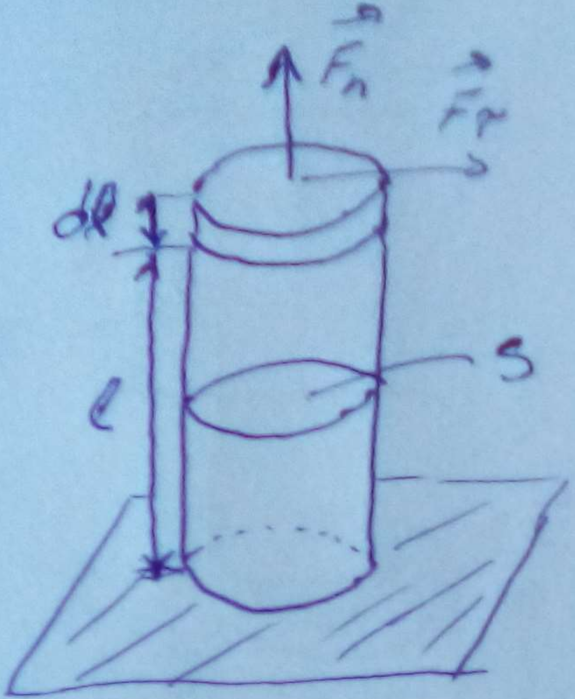
e - ексцентрицитет.



$$\left.\begin{array}{l} e=0 \text{ кръг} \\ 0<e<1 \text{ елипса} \end{array}\right\} E<0$$

$$\left.\begin{array}{l} e=1 \text{ парабола} \\ e>1 \text{ хипербола} \end{array}\right\} E>0$$

Хуков закон



$$\sigma_n = \frac{F_n}{S}$$

$$\sigma_\tau = \frac{F_\tau}{S}$$

$$\delta = \frac{\Delta V}{V} = \frac{\Delta l \cdot S}{l \cdot S}$$

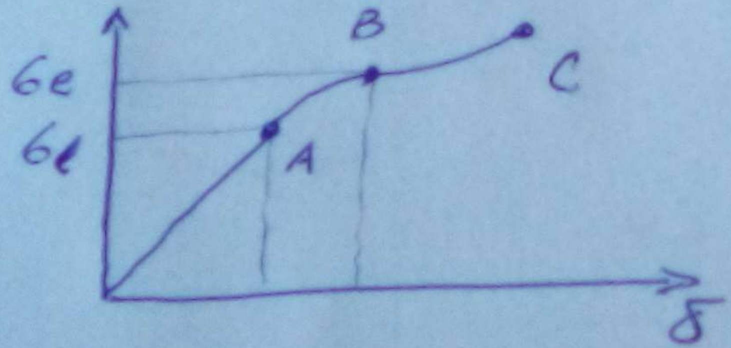
$$\boxed{\sigma = E_Y \cdot \delta}$$

E_Y - Јангов модул еластичности.

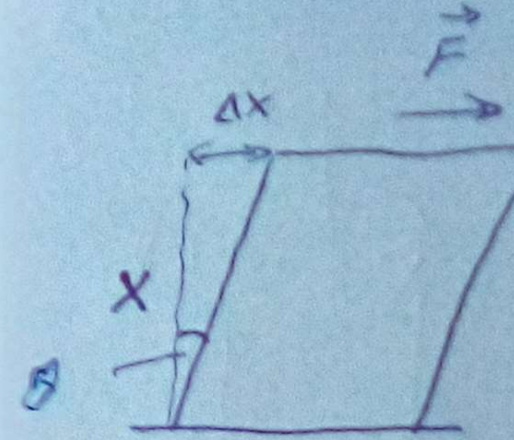
$$\frac{F}{S} = E_Y \cdot \frac{\Delta l}{l}$$

$$F = \underbrace{\frac{E_Y \cdot S}{l}}_{= k} \cdot \Delta l$$

Дијаграм напона



Смицање и торзија

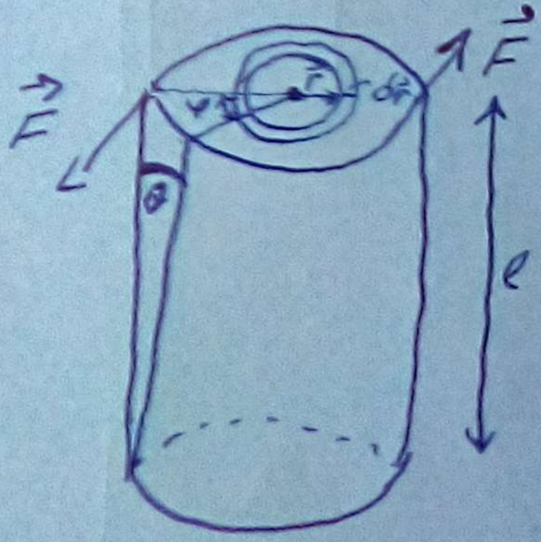


$$\sigma_\tau = \frac{F}{S}$$

$$\frac{\Delta x}{x} = \operatorname{tg}\theta \approx \theta$$

$$\sigma_\tau = \frac{F}{S} = G\cdot\frac{\Delta x}{x} = G\cdot\theta$$

G - модул смицања



Торзија

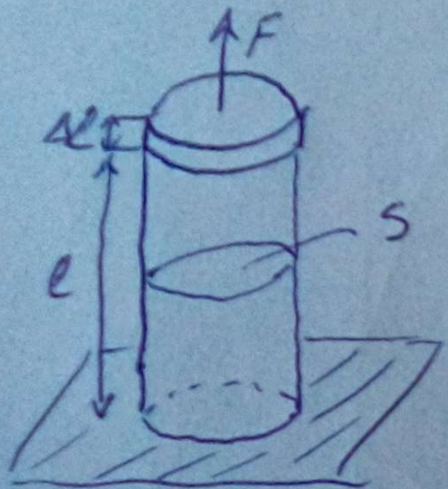
$$\sigma_\tau = \frac{dF}{dS} = \underbrace{\frac{dF}{2\pi r\,dr} = G\cdot\frac{r\cdot\varphi}{l}} = G\cdot\theta$$

$$\Rightarrow \quad dF = G\cdot\frac{2\pi r^2 dr}{l}\cdot\varphi \quad / \cdot r$$

$$dM = r\cdot dF = \frac{2\pi G\cdot r^3 dr}{l}\cdot\varphi$$

$$M = \frac{2\pi G}{l}\left(\int_0^R r^3 dr\right)\cdot\varphi = \underbrace{\left(\frac{\pi G\cdot R^4}{2l}\right)}_{C}\cdot\varphi = C\cdot\varphi$$

Енергија еластичне деформације



$$dA = F \cdot d(\Delta l) = E_Y \cdot S \cdot \delta \, d(\Delta l) = E_Y \cdot S \cdot \frac{\Delta l \, d(\Delta l)}{l}$$

$$A = E_Y \cdot S \cdot \frac{1}{l} \int_0^{\Delta l} \Delta l \, d(\Delta l) = E_Y \cdot S \frac{\Delta l^2}{2l}$$

$$A = U = \frac{1}{2} \cdot E_Y \cdot S \cdot l \cdot \frac{\Delta l^2}{l^2} = E_Y \cdot \frac{V}{2} \delta^2$$

$$\rho_U = \frac{U}{V} = \frac{1}{2} E_Y \cdot \delta^2$$