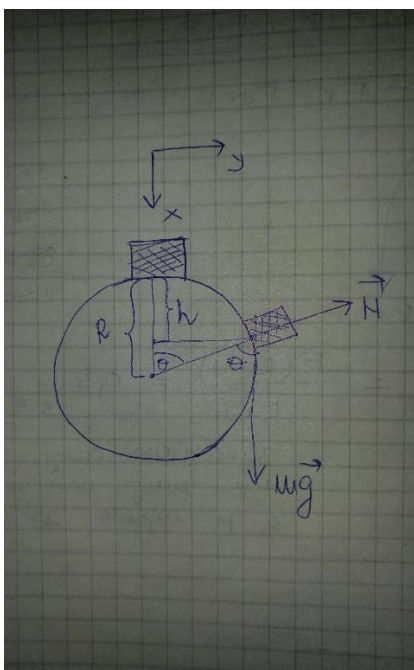


ZADACI (TERMIN 8. MAJ 2020.)

1. Malo telo klizi sa vrha lopte naniže. Sa koje visine h u odnosu na vrh će se telo odvojiti od površine lopte i poleteti naniže?
2. Malo telo klizi niz strmu ravan koja prelazi u mrtvu petlju čiji je poluprečnik R . Sa koje visine h_1 će pasti telo ako mu je početna visina jednaka h ? Kolika treba da bude visina h da bi telo načinilo potpunu petlju u vertikalnoj ravni ne padajući naniže?
3. Pločica mase $m = 1$ kg leži na horizontalnoj podlozi. Udarcem joj je, u početnom trenutku, data brzina od 1.5 m/s. Koeficijent trenja između pločice i podloge je $\mu = 0.27$. Odrediti srednju snagu sile trenja za vreme kretanja pločice.
4. Nagib jednog dela puta jednak je 0.05 . Spuštajući se po nagibu, pri isljučenom motoru, automobil se kreće ravnomerno brzinom od 60 km/h. Kolika mora biti snaga automobila da bi se kretao na istom takvom usponu jednakom brzinom. Masa automobila je 1.5 t.
5. Automobil se kreće uzbrdo, po malom nagibu, stalnom brzinom od 3 m/s. Ako se on kreće u obratnom smeru tj nizbrdo, ali sa jednaom snagom motora postiže brzinu od 7 m/s. Izračunati brzinu koju će za istu snagu imati motor pri kretanju po horizontalnom putu? (Pretpostaviti da vučna sila ne zavisi od brzine)
6. Na konopcima visi sanduk sa peskom sa ravnom gornjom površinom. Za sanduk je privezan dugačak kanap čiji je slobodan kraj provučen ispod kartona koji je pričvršćen za slobodan stativ tako da se kanap nalazi u horizontalnom položaju. Rastojanje od gornje površine sanduka do tačke oslonca je poznato i iznosi l . Revolverom se gađa sanduka sa bočne strane zatim sanduk skrene povlačeći kanap ispod kartona za dužinu s , znatno manju od rastojanja l . Uzimajući u obzir da je masa metka m_1 u odnosu na masu sanduka m_2 mala, dokazati da se brzina metka može izračunati po formuli $v = \frac{m_1}{m_2} * s * \sqrt{\frac{g}{l}}$.

Resenja:

1. Ključni delovi postavke zadatka su: primena II Njutnovog zakona na sistem (dakle projektovati sve sile koje deluju na telo ali u odnosu na orjentisani Dekartov koordinatni sistem), zatim primena zakona održanja energije i određivanje položaja tela u odnosu na početni trenutak (visina sa koje će telo pasti po uslovu zadatka).



$$\begin{aligned} (1) \quad u &= R - R \cos \theta = R(1 - \cos \theta) \\ (2) \quad \frac{mv^2}{R} &= -N + mg \cos \theta \\ (3) \quad \frac{mv^2}{2} &= mgl \end{aligned}$$

Projekcija vektora položaja pločice na x osu Dekartovog sistema.

II Njutnov zakon primenjen na pločicu na lopti

Trenutni položaj pločice u odnosu na početni položaj

Zakon održanja energije

$$\begin{aligned} (2) \quad (3) \Rightarrow \frac{mv^2}{2} &= mgR(1 - \cos \theta) \\ \frac{mv^2}{R} &= 2mg(1 - \cos \theta) \\ mg \cos \theta - N &= 2mg(1 - \cos \theta) \\ \cos \theta &= 2 - 2 \cos \theta \\ \cos \theta (1 + 2) &= 2 \\ \cos \theta &= \frac{2}{3} \\ u &= R(1 - \frac{2}{3}) = \frac{1}{3} R \end{aligned}$$

$$\begin{aligned} \# \quad T - T_0 &= U_0 - U \\ \frac{mv^2}{2} &= + mgl = R - u \\ &= mgl - mgl \cos \theta \\ &= mgl(1 - \cos \theta) = \\ &= mgl \end{aligned}$$

ova jednačina je zakazana za yenu napredno

2.

Primena zakona održanja energije na sistem.

a) $h' = h_1 = ?$

(1) $h' = R(1 - \cos \theta)$

(2) $\frac{mv^2}{R} = H - mg \cos \theta$

(3) $\frac{mgL}{2} = mg(h - h')$

(1) (3) $\Rightarrow$

$$\frac{mv^2}{2} = mg(h - R(1 - \cos \theta))$$

$$\frac{mv^2}{2} = mgR \left(\frac{L}{R} - (1 - \cos \theta) \right)$$

$$\frac{mv^2}{R} = 2mg \left(\frac{L}{R} - (1 - \cos \theta) \right)$$

$$N - mg \cos \theta = 2mg \left(\frac{L}{R} - (1 - \cos \theta) \right)$$

$$2mg \frac{L}{R} - 2mg + 2mg \cos \theta = -mg \cos \theta$$

$$2mg \left(\frac{L}{R} - 1 \right) = \cos \theta (-mg - 2mg)$$

$$2/g \left(\frac{L}{R} - 1 \right) = -3 \cos \theta$$

$$\frac{2L}{R} - 2 = -3 \cos \theta$$

Kombinacija prve i treće jednačine.

Prva jednačina (položaj tela u odnosu na horizontalnu podlogu).

$$3\cos\theta = 2 - 2\frac{h}{R}$$

$$\cos\theta = \frac{2}{3} \left(1 - \frac{h}{R}\right)$$

Zamenom ugla u prvu jednačinu dobija se tražena visina.

$$\begin{aligned} \text{(A)} \Rightarrow h &= R \left(1 - \frac{3}{2} \left(1 - \frac{h}{R}\right)\right) = \\ &= R - \frac{3}{2}R + \frac{3}{2}h = \\ &= R + \frac{3}{2}(h - R) \end{aligned}$$

$$\delta) h' = 2R$$

Rastojanje između tela i horizontalne podloge u trenutku kada telo načini punu petlju (kada se nađe na rastojanju $2R$ od horizontalne podloge)

$$2R = R + \frac{2}{3}(h - R)$$

$$R = +\frac{2}{3}h - \frac{2}{3}R$$

$$R + \frac{2}{3}R = +\frac{2}{3}h$$

$$\frac{5R}{3} = +\frac{2}{3}h$$

$$5R = 2h \Rightarrow h = \frac{5R}{2}$$

3.

$M = 0,27$
 $v_0 = 1,5 \text{ m/s}$

$\mu \frac{d^2x}{dt^2} = -\mu g$
 $\frac{dv}{dt} = -\mu g$
 $x(t) = -\mu g \frac{t^2}{2} + v_0 t$

$\vec{P} = \frac{1}{\tau} \int_0^\tau \vec{F} \cdot \vec{v} dt = \frac{1}{\tau} \int_0^\tau (-\mu mg)(v_0 - \mu gt) dt =$

$\# \frac{dx(\tau)}{dt} = 0 \Rightarrow -\mu g \tau + v_0 = 0$
 $-\mu g \tau = -v_0$
 $\tau = \frac{v_0}{\mu g}$

$\# v = \frac{ds}{dt} \Rightarrow a = \frac{dv}{dt}$
 $ds = v dt \Rightarrow dv = a dt$

$= \frac{1}{\tau} (-\mu mg) v_0 \tau + \frac{1}{\tau} (\mu mg) \cdot \mu g \frac{\tau^2}{2} = -\mu mg v_0 + \frac{\mu^2 g^2 \tau^2}{2} =$
 $= -\mu mg v_0 + \frac{\mu g v_0^2}{2} = -\frac{\mu g v_0^2}{2}$

Usrednjena snaga po vremenskom intervalu kretanja tela.

Krajnja brzina kretanja tela kada se telo zaustavi je nula što koristimo kako bismo došli do ukupnog vremena kretanja tela.

4.

Ravnomerno kretanje tela po uslovu zadatka.

The diagram shows a block of mass m on an inclined plane with height H and length L . The angle of the incline is α . Forces acting on the block are gravity mg , normal force N , friction force F_{tr} , and a pulling force F_v along the incline. The coordinate system has x along the incline and y perpendicular to it.

Equations derived from Newton's second law:

$$\begin{aligned} (1) \quad m \frac{d^2 x}{dt^2} &= mg \sin \alpha - \mu N \\ m \frac{d^2 y}{dt^2} &= -mg \cos \alpha + N \\ \Rightarrow N &= mg \cos \alpha \\ (2) \quad m \frac{d^2 x}{dt^2} &= F_v - F_{tr} - mg \sin \alpha \\ m \frac{d^2 x}{dt^2} &= F_v - \mu mg \cos \alpha - mg \sin \alpha \\ \Rightarrow F_v &= \mu mg \cos \alpha + mg \sin \alpha = mg (\mu \cos \alpha + \sin \alpha) \end{aligned}$$

Power calculation:

$$P = F_v \cdot v = mgv (\mu \cos \alpha + \sin \alpha) = 2mgv \sin \alpha$$

Velocity u is related to height H and length L by:

$$u = \frac{H}{L} = \frac{1}{2} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\sin \alpha}{\sqrt{1 - \sin^2 \alpha}}$$

$$\sin \alpha = u \sqrt{1 - \sin^2 \alpha}$$

$$\sin^2 \alpha = u^2 (1 - \sin^2 \alpha)$$

$$\sin^2 \alpha + u^2 \sin^2 \alpha = u^2$$

$$\sin^2 \alpha (1 + u^2) = u^2$$

$$\sin^2 \alpha = \frac{u^2}{1 + u^2} \Rightarrow \sin \alpha = \frac{u}{\sqrt{1 + u^2}}$$

Final power calculation:

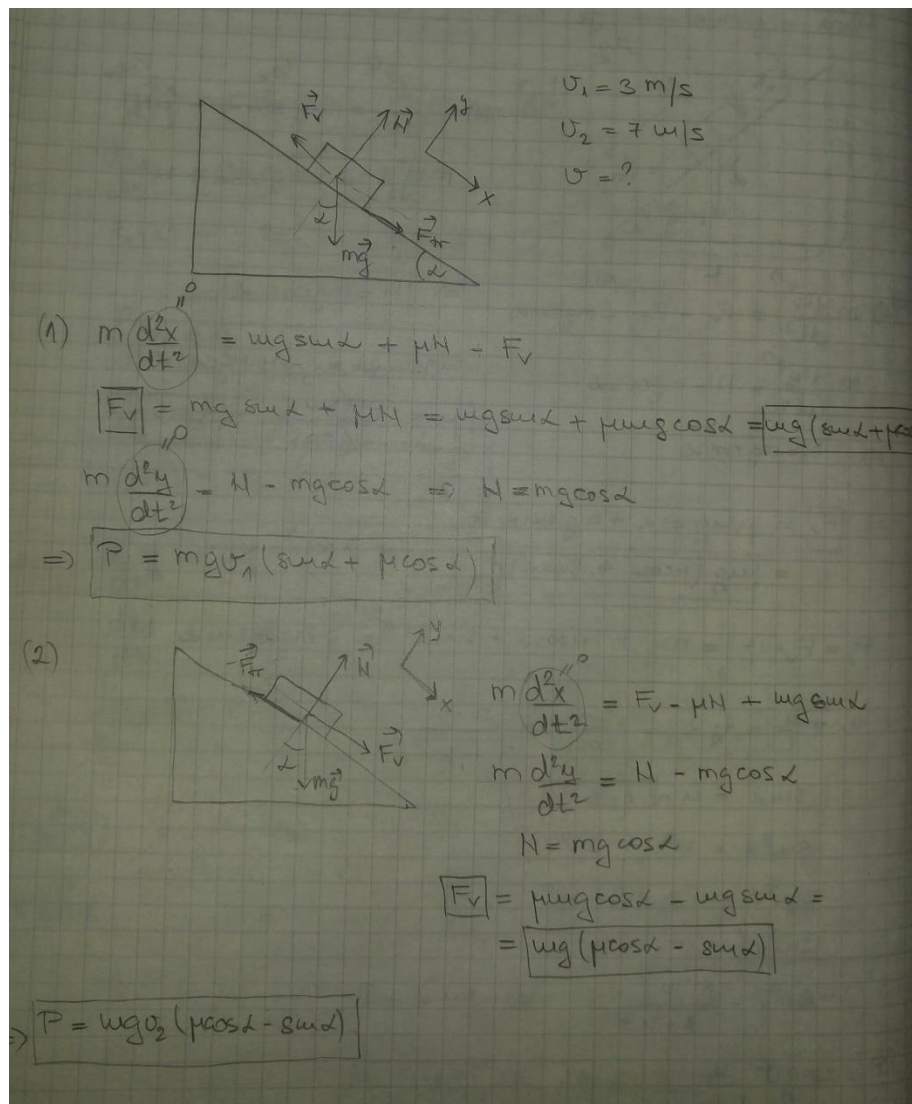
$$P = 2mgv \frac{u}{\sqrt{1 + u^2}} = 24494,4011 \text{ W}$$

Kretanje niz strmu ravan.

Vučna sila motora.

Kretanje uz strmu ravan.

5.



$v_1 = 3 \text{ m/s}$
 $v_2 = 7 \text{ m/s}$
 $v = ?$

(1)

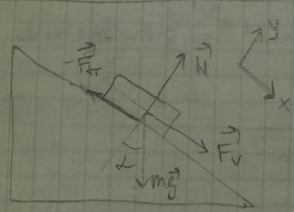
$$m \frac{d^2 x}{dt^2} = \mu g \sin \alpha + \mu N - F_v$$

$$F_v = mg \sin \alpha + \mu N = \mu g \sin \alpha + \mu mg \cos \alpha = \mu g (\sin \alpha + \mu \cos \alpha)$$

$$m \frac{d^2 y}{dt^2} = N - mg \cos \alpha \Rightarrow N = mg \cos \alpha$$

$$\Rightarrow P = mg v_1 (\sin \alpha + \mu \cos \alpha)$$

(2)



$$m \frac{d^2 x}{dt^2} = F_v - \mu N + \mu g \sin \alpha$$

$$m \frac{d^2 y}{dt^2} = N - mg \cos \alpha$$

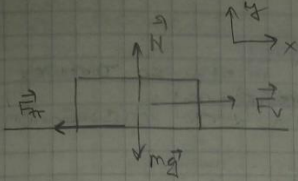
$$N = mg \cos \alpha$$

$$F_v = \mu mg \cos \alpha - \mu g \sin \alpha = \mu g (\mu \cos \alpha - \sin \alpha)$$

$$\Rightarrow P = \mu g v_2 (\mu \cos \alpha - \sin \alpha)$$

Fakultet za Fizicku Hemiju
 Predmet: Fizika 1
 Tip vezbi: Racunske vezbe
 Asistent: Violeta Stankovic
 Datum: 8. maj 2020.

3°



$$m \frac{d^2 x}{dt^2} = F_v - \mu N$$

$$m \frac{d^2 y}{dt^2} = N - mg$$

$$\Rightarrow N = mg$$

$$\Rightarrow F_v = \mu mg$$

$$\Rightarrow P = \mu mg v$$

$$v = \frac{P}{\mu mg}$$

$\mu = ?$

$$v_1 \cancel{\mu g} (\sin \alpha + \mu \cos \alpha) = v_2 \cancel{\mu g} (\mu \cos \alpha - \sin \alpha)$$

$$v_1 \sin \alpha + v_1 \mu \cos \alpha = v_2 \mu \cos \alpha - v_2 \sin \alpha$$

$$\mu (v_1 \cos \alpha - v_2 \cos \alpha) = -v_1 \sin \alpha - v_2 \sin \alpha \quad / \cdot (-1)$$

$$\mu \cos \alpha (v_2 - v_1) = \sin \alpha (v_1 + v_2)$$

$$\mu = \frac{\sin \alpha (v_1 + v_2)}{\cos \alpha (v_2 - v_1)}$$

$$v = \frac{mg v_1 (\sin \alpha + \frac{\sin \alpha \cos \alpha (v_1 + v_2)}{\cos \alpha (v_2 - v_1)})}{mg \frac{\sin \alpha (v_1 + v_2)}{\cos \alpha (v_2 - v_1)}} = \frac{v_1 \cos \alpha (v_2 - v_1) [\sin \alpha + \sin \alpha \frac{v_1 + v_2}{(v_2 - v_1)}]}{\sin \alpha (v_1 + v_2)}$$

$$= \frac{v_1 \cos \alpha [\sin \alpha v_2 - \cancel{\sin \alpha v_1} + \sin \alpha v_1 + \sin \alpha v_2]}{\sin \alpha (v_1 + v_2)}$$

$$= \frac{2 v_1 \cos \alpha \sin \alpha v_2}{\sin \alpha (v_1 + v_2)} = \frac{2 v_1 v_2 \cos \alpha}{v_1 + v_2} \approx 4.2 \frac{m}{s}$$

$\approx 1 \leftarrow$ μ odgovara je petkom
ga je nam namo

6.

The image shows a handwritten physics solution on grid paper. At the top, there are two diagrams. The left diagram shows a pendulum of length l and mass m_1 at an angle, with a vertical dashed line indicating its initial position. The right diagram shows a projectile of mass m_2 moving horizontally with velocity v towards a sandbag of mass m_1 hanging from a string of length l . The height of the sandbag is h . A right-angled triangle is drawn with hypotenuse l , horizontal side s , and vertical side h . The equation $h = l - \sqrt{l^2 - s^2}$ is written below the triangle.

The solution proceeds with the following steps:

- (1) $m_1 v = (m_1 + m_2) v'$ 30H
- (2) $\frac{1}{2} (m_1 + m_2) v'^2 = (m_1 + m_2) g h \Rightarrow v' = \sqrt{2gh}$

Then, the velocity v' is expressed in terms of the initial velocity v :

$$v' = \frac{m_1 + m_2}{m_1} \sqrt{2gh} =$$

$$= \frac{m_1 + m_2}{m_1} \sqrt{2g(l - \sqrt{l^2 - s^2})} = \frac{m_1 + m_2}{m_1} \sqrt{2g} (l - (l^2 - s^2)^{1/2})^{1/2}$$

A note "Razvoj u red." (Expansion in series) points to the term $(l^2 - s^2)^{1/2}$. The expansion is shown as:

$$(l^2 - s^2)^{1/2} = l \left(1 - \left(\frac{s}{l}\right)^2\right)^{1/2} \approx l \left(1 - \frac{1}{2} \frac{s^2}{l^2}\right) = l - \frac{1}{2} \frac{s^2}{l}$$

Substituting this back into the expression for v' :

$$v' \approx \frac{m_1 + m_2}{m_1} \sqrt{2g} \left(l - l + \frac{1}{2} \frac{s^2}{l}\right)^{1/2} \approx \frac{m_1 + m_2}{m_1} \sqrt{2g} \left(\frac{1}{2} \frac{s^2}{l}\right)^{1/2}$$

$$\approx \frac{m_2}{m_1} s \left(\frac{g}{l}\right)^{1/2} = \frac{m_2}{m_1} s \left(\frac{g}{l}\right)^{1/2}$$

Three text boxes with arrows pointing to specific parts of the work:

- "Zakon održanja energije." points to the energy conservation equation (2).
- "Brzina nakon sudara metka i sanduka." points to the final expression for v' .
- "Razvoj u red." points to the expansion of $(l^2 - s^2)^{1/2}$.