

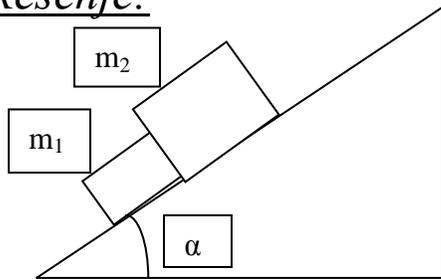
# Predmet: Fizika 1

## Računske vežbe (Termin: 3.4.2020)

Predmetni asistent na računskim vežbama: Violeta Stanković

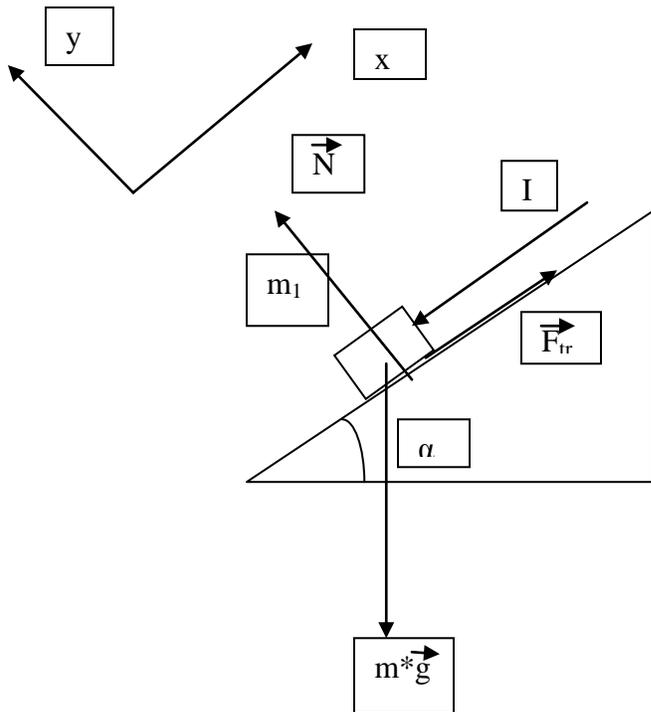
1. Na strmu ravan nagiba  $\alpha$  postavljene su jedna uz drugu dve ploče. Mase ploča su  $m_1$  i  $m_2$ , a koeficijenti trenja između ploča i podloge su  $\mu_1$  i  $\mu_2$  ( $\mu_1 > \mu_2$ ). Odrediti:
- Silu interakcije ploča u kretanju
  - Minimalnu vrednost ugla  $\alpha$  pri kojoj počinje klizanje

Rešenje:



I=?

Primenom Drugog Njutnovog zakona na telo mase  $m_1$ , sve sile koje deluju na telo projektuju se na dve ose dekartovog sistema (u odnosu na koji posmatramo kretanje). Identičan postupak se primenjuje na drugo telo datog sistema.



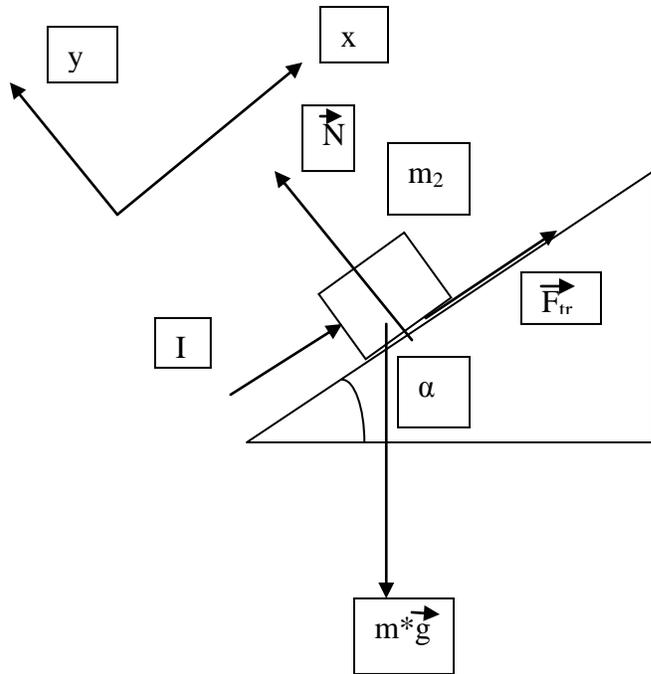
Handwritten solution on graph paper:

$$m_1; m_2$$
$$M_1; M_2 \quad (\mu_1 > \mu_2)$$
$$I = ?$$
$$m_1 mg \cos \alpha$$
$$(A) \quad m \frac{d^2 x_1}{dt^2} = -m g \sin \alpha + \overset{N}{M_1 H} - I$$
$$m \frac{d^2 y_1}{dt^2} = 0 = N - m g \cos \alpha \Rightarrow N = m g \cos \alpha$$

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$$m_2 \frac{d^2 x_2}{dt^2} = -m_2 g \sin \alpha + M_2 N + I$$
$$m_2 \frac{d^2 y_2}{dt^2} = N - m_2 g \cos \alpha$$
$$\Rightarrow N = m_2 g \cos \alpha$$

$$\Rightarrow x_2 = x_1 + L \quad \frac{d^2}{dt^2}$$
$$-m_1 g \sin \alpha + M_1 g \cos \alpha - \frac{I}{m_1} = -\frac{m_2 g \sin \alpha}{m_2} + \frac{M_2 m_2 g \cos \alpha}{m_2} + \frac{I}{m_2}$$
$$M_1 g \cos \alpha - \frac{I}{m_1} = M_2 g \cos \alpha + \frac{I}{m_2}$$
$$-I \left( \frac{1}{m_1} + \frac{1}{m_2} \right) = M_2 g \cos \alpha - M_1 g \cos \alpha$$
$$I \left( \frac{m_1 + m_2}{m_1 m_2} \right) = g \cos \alpha (M_1 - M_2)$$
$$I = \frac{g \cos \alpha (M_1 - M_2)}{\left( \frac{m_1 + m_2}{m_1 m_2} \right)}$$

Da bismo došli do tražene sile  $I$ , koristićemo činjenicu da se  $x$  koordinata drugog tela može napisati kao zbir  $x$  koordinata prvog tela i dužine prvog tela. Diferenciranjem pomenutog izraza se dobija da je ubrzanje prvog tela jednako ubrzanju drugog tela.

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$$\begin{aligned} \frac{d^2 x}{dt^2} &= -g \sin \alpha + \mu_2 g \cos \alpha + \frac{g \cos \alpha (\mu_1 - \mu_2)}{\frac{m_1 + m_2}{m_1 m_2}} = \\ &= g \left[ -(m_1 + m_2) \sin \alpha + \underbrace{\mu_2 \cos \alpha (m_1 + m_2)}_{\mu_2 \cos \alpha m_1 + \mu_2 \cos \alpha m_2} + \frac{\cos \alpha (\mu_1 - \mu_2) m_1}{\cos \alpha \mu_1 m_1 - \cos \alpha \mu_2 m_1} \right] \frac{1}{m_1 + m_2} = \\ &= \frac{g}{m_1 + m_2} \left[ -(m_1 + m_2) \sin \alpha + \cos \alpha (\mu_1 m_1 + \mu_2 m_2) \right] \end{aligned}$$

$\frac{d^2 x}{dt^2} \leq 0$  - kada ce ueno kugla (jep je odučen okrenuti ka tope, a ne ka gore)

$$\frac{g}{m_1 + m_2} \left[ -(m_1 + m_2) \sin \alpha + \cos \alpha (\mu_1 m_1 + \mu_2 m_2) \right] \leq 0$$
$$-(m_1 + m_2) \sin \alpha \leq -(\mu_1 m_1 + \mu_2 m_2) \cos \alpha \quad / \cdot (-1)$$
$$\frac{1}{g} \alpha \geq \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}$$

$$\alpha \geq \arcsin \left( \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2} \right)$$

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2. Prizma na kojoj se nalazi pločica mase  $m$  kreće se po horizontalnoj podlozi ubrzanjem  $w$ . Kolika je maksimalna vrednost  $w$  za koju će pločica ostati nepokretna u odnosu na prizmu ako se zna da je koeficijent trenja između njih  $\mu < \text{ctg } \alpha$ .

$$m \frac{d^2x}{dt^2} = -mgsin\alpha - \mu N + m w \cos\alpha$$
$$m \frac{d^2y}{dt^2} = -mg \cos\alpha + N - m w \sin\alpha$$
$$N = m(g \cos\alpha + w \sin\alpha)$$
$$F_{tr} \leq \mu N$$
$$F_{tr} \leq \mu m(g \cos\alpha + w \sin\alpha)$$
$$\Rightarrow \mu (w \cos\alpha - g \sin\alpha) \leq \mu m(g \cos\alpha + w \sin\alpha)$$
$$w \cos\alpha - g \sin\alpha \leq \mu g \cos\alpha + \mu w \sin\alpha$$

Projekcijom sila na x i y osu dolazimo do sile trenja koja je u ovom slučaju sila od interesa.

Zatim koristimo uslov zadatka da pločica treba da ostane nepokretna u odnosu na prizmu, ali uslov prilagođavamo orijentaciji Dekartovog koordinatnog sistema.

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Handwritten derivation on grid paper:

$$W \cos \alpha - \mu W \sin \alpha \leq \mu g \cos \alpha + g \sin \alpha$$
$$W (\cos \alpha - \mu \sin \alpha) \leq g (\mu \cos \alpha + \sin \alpha)$$

$$W \leq \frac{g (\mu \cos \alpha + \sin \alpha)}{\cos \alpha - \mu \sin \alpha}$$

$$\Rightarrow W \leq \frac{g (\mu \sin \alpha + 1)}{\sin \alpha - \mu}$$

Diagram showing a vertical axis labeled  $F_{tr}$  with a tick mark for  $F_{tr}^{smax}$ . Horizontal arrows point towards the axis from both sides. A horizontal axis labeled  $g$  is shown below.

### 3. Nekoliko strmih ravni imaju zajedničku osnovu.

- Koliki je nagib strme ravni prema horizontu ako je vreme spuštanja tela po toj strmoj ravni kraće nego po ostalim. (Razmotriti slučajeve kada je trenje zanemarljivo I kada je koeficijent trenja  $\mu=0.25$ .)
- Koliki je koeficijent trenja ako su vremena spuštanja pri nagibu strme ravni za  $\alpha_1 = 60^\circ$  i  $\alpha_2 = 45^\circ$  jednaka?

Handwritten diagram and equations for a block on an inclined plane:

Diagram shows a block on an inclined plane with angle  $\alpha$ . Forces acting on the block are: normal force  $\vec{N}$  perpendicular to the plane, weight  $\vec{mg}$  vertically downwards, and friction force  $\vec{F}_{tr}$  up the plane. The coordinate system  $(x, y)$  is defined with  $x$  along the plane and  $y$  perpendicular to it.

$$m \frac{d^2 x}{dt^2} = mg \sin \alpha - F_{tr}$$
$$m \frac{d^2 y}{dt^2} = N - mg \cos \alpha$$
$$\Rightarrow N = mg \cos \alpha$$

Additional note:  $\mu N = \mu mg \cos \alpha$

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$$\frac{d^2x}{dt^2} = g \sin \alpha - \mu g \cos \alpha$$
$$\frac{dx}{dt} = g \sin \alpha t - \mu g \cos \alpha t + \vec{v}_1^0$$
$$x(t) = g \sin \alpha \frac{t^2}{2} - \mu g \cos \alpha \frac{t^2}{2} + \vec{v}_2^0 =$$
$$= g \frac{t^2}{2} (\sin \alpha - \mu \cos \alpha)$$
$$s(t) = x(t) \quad \left\{ \begin{array}{l} \frac{l}{\cos \alpha} = \frac{g t^2}{2} (\sin \alpha - \mu \cos \alpha) \\ s = \frac{l}{\cos \alpha} \end{array} \right.$$
$$t^2 = \frac{2l}{g \cos \alpha (\sin \alpha - \mu \cos \alpha)}$$
$$t^2 = t^2(x)$$
$$t^2(\alpha_0) = \min t^2(\alpha_0) \Rightarrow \frac{d t^2(\alpha_0)}{d \alpha} = 0$$
$$\frac{d t^2}{d \alpha} = \frac{-2l \left[ \sin \alpha_0 (\sin \alpha_0 - \mu \cos \alpha_0) + \cos \alpha_0 (\cos \alpha_0 + \mu \sin \alpha_0) \right] g}{g^2 \cos^2 \alpha_0 (\sin \alpha_0 - \mu \cos \alpha_0)^2} = 0$$
$$2lg \left[ \sin^2 \alpha_0 - \mu \sin \alpha_0 \cos \alpha_0 - \cos^2 \alpha_0 - \mu \cos \alpha_0 \sin \alpha_0 \right] = 0$$
$$\sin^2 \alpha_0 - \cos^2 \alpha_0 - 2\mu \sin \alpha_0 \cos \alpha_0 = 0 \quad \left| \frac{1}{\cos^2 \alpha_0} \right.$$
$$\frac{1}{2} \alpha_0 - 1 - 2\mu \frac{\sin \alpha_0}{\cos \alpha_0} = 0$$
$$\# \frac{1}{2} \alpha = \frac{2 \frac{1}{2} \alpha}{1 - \frac{1}{2} \alpha} \quad \#$$
$$-2\mu \frac{1}{2} \alpha = 1 - \frac{1}{2} \alpha$$
$$2 \frac{1}{2} \alpha = \left( \frac{1}{\mu} \right) (1 - \frac{1}{2} \alpha)$$
$$\frac{2 \frac{1}{2} \alpha}{(1 - \frac{1}{2} \alpha)} = -\frac{1}{\mu}$$
$$\# \frac{1}{2} \alpha = \frac{2 \frac{1}{2} \alpha}{1 - \frac{1}{2} \alpha} \quad \#$$

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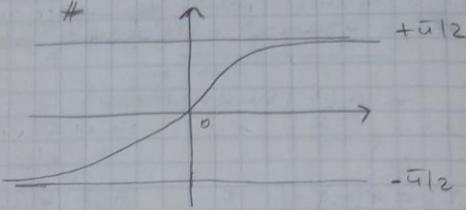
$\operatorname{tg} 2\alpha_0 = -\frac{1}{\mu}$

$2\alpha_0 = \operatorname{arctg}\left(-\frac{1}{\mu}\right)$

$\alpha_0 = \frac{1}{2} \operatorname{arctg}\left(-\frac{1}{\mu}\right)$

$\alpha_0 = 45^\circ$       $\square$

$\alpha_0 = 52^\circ$



δ)  $\Gamma^2(\alpha_1) = \Gamma^2(\alpha_2)$

$$\frac{2b}{g \cos \alpha_1 (\sin \alpha_1 - \mu \cos \alpha_1)} = \frac{2b}{g \cos \alpha_2 (\sin \alpha_2 - \mu \cos \alpha_2)}$$
$$\cos \alpha_1 \sin \alpha_1 - \mu \cos^2 \alpha_1 = \cos \alpha_2 \sin \alpha_2 - \mu \cos^2 \alpha_2$$
$$\mu (\cos^2 \alpha_2 - \cos^2 \alpha_1) = \cos \alpha_2 \sin \alpha_2 - \cos \alpha_1 \sin \alpha_1$$

$$\mu = \frac{\cos \alpha_2 \sin \alpha_2 - \cos \alpha_1 \sin \alpha_1}{\cos^2 \alpha_2 - \cos^2 \alpha_1} = 0,267549$$

#  $\pm = \operatorname{tg} \alpha_0$      2°  $\mu = 0$

$$\pm^2 - 2\mu \pm - 1 = 0$$
$$\pm_{1,2} = \frac{2\mu \pm \sqrt{4\mu^2 + 4}}{2} = \pm = 1$$
$$= \mu \pm \sqrt{\mu^2 + 1}$$

$\operatorname{tg} \alpha_0 = 1$

$\alpha_0 = \operatorname{arctg} 1 = 45^\circ$

$$\Rightarrow \pm_1 = \mu + \sqrt{\mu^2 + 1} = \frac{1}{4} + \sqrt{\frac{17}{16}} =$$
$$= \frac{1 + \sqrt{17}}{4} = 52^\circ$$

$\pm_2$  - tuje dodatni moji te pemebe  
jep ce govija reaktivan ugao  $\alpha_0$

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4. Nagib jednog dela puta iznosi 0.05. Spuštajući se po nagibu pri isključenom motoru automobil se kreće ravnomerno brzinom od 50 km/h. Koliko bi iznosila vučna sila motora ako bi se automobil kretao po istom takvom usponu jednakom brzinom? (Masa automobila je 1t)

The image shows a handwritten solution on grid paper. It consists of two diagrams of a car on an incline of angle  $\alpha$ . The first diagram shows the car moving down the incline with forces  $F_{tr}$  (traction),  $N$  (normal force), and  $mg$  (gravity). The second diagram shows the car moving up the incline with forces  $F_v$  (traction),  $N$  (normal force), and  $mg$  (gravity). To the right of the diagrams are Newton's second law equations for both cases, leading to the derivation of the coefficient of friction  $\mu = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha$ . Below the diagrams, the calculation for the traction force  $F_v$  is shown, resulting in  $F_v = 2 \sin \alpha mg = \frac{2u}{\sqrt{1+u^2}} mg = 9,945 \text{ N}$ . On the left side, there are trigonometric derivations for  $\sin \alpha$  based on the velocity  $u$  and the incline angle  $\alpha$ .

1)  $m \frac{d^2 x}{dt^2} = -\mu N + mg \sin \alpha$   
2)  $m \frac{d^2 y}{dt^2} = N - mg \cos \alpha$   
2)  $\Rightarrow N = mg \cos \alpha$   
1)  $\Rightarrow \mu mg \cos \alpha = mg \sin \alpha$   
 $\mu = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha$

1)  $m \frac{d^2 x}{dt^2} = +\mu N - F_v + mg \sin \alpha$   
2)  $m \frac{d^2 y}{dt^2} = N - mg \cos \alpha$   
2)  $\Rightarrow N = mg \cos \alpha$   
1)  $\Rightarrow F_v = \mu N + mg \sin \alpha =$   
 $= \mu mg \cos \alpha + mg \sin \alpha =$   
 $= mg (\mu \cos \alpha + \sin \alpha) =$   
 $= mg \left( \frac{\sin \alpha}{\cos \alpha} + \sin \alpha \right) =$   
 $= 2 \sin \alpha mg = \frac{2u}{\sqrt{1+u^2}} mg$   
 $= 9,945 \text{ N}$

$u = \frac{v}{L \cos \alpha}$   
 $\sin \alpha = u \cos \alpha$   
 $= u \sqrt{1 - \sin^2 \alpha}$   
 $\sin^2 \alpha = u^2 (1 - \sin^2 \alpha)$   
 $\sin^2 \alpha + u^2 \sin^2 \alpha = u^2$   
 $\sin^2 \alpha (1 + u^2) = u^2$   
 $\sin^2 \alpha = \frac{u^2}{1 + u^2}$

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5. Malo telo kreće se niz strmu ravan nagibnog ugla  $\alpha$ . Koeficijent trenja zavisi od pređenog puta  $x$  kao  $\mu = a \cdot x$ , gde je  $a$  konstanta. Naći put koji telo pređe do zaustavljanja i maksimalnu brzinu kretanja duž datog puta.

Diagram: A block on an inclined plane with angle  $\alpha$ . Forces shown:  $F_H$  (friction),  $N$  (normal force),  $mg$  (gravity), and  $H$  (normal force vector). Coordinate system  $x$  is along the incline.

Given:  $\mu = ax$   
 $s = ?$   
 $v_{max} = ?$

Equations of motion:  
 $m \frac{d^2x}{dt^2} = mg \sin \alpha - \mu N$   
 $m \frac{d^2y}{dt^2} = N - mg \cos \alpha$   
 $N = mg \cos \alpha$

Substituting  $\mu = ax$  and  $N = mg \cos \alpha$  into the first equation:  
 $m \frac{d^2x}{dt^2} = mg \sin \alpha - \frac{ax^2}{\cos \alpha}$   
 $a_x = g \sin \alpha - \frac{ax^2}{\cos \alpha}$

Velocity as a function of position:  
 $\frac{dv}{dx} \cdot \frac{dx}{dt} = g \sin \alpha - \frac{ax^2}{\cos \alpha}$   
 $v dv = g \sin \alpha dx - \frac{ax^2}{\cos \alpha} dx$   
 $\int_0^v v dv = \int_0^x (g \sin \alpha - \frac{ax^2}{\cos \alpha}) dx$   
 $\frac{v^2}{2} = g \sin \alpha x - \frac{ax^3}{3 \cos \alpha}$

Maximum velocity when  $v = 0$ :  
 $0 = g \sin \alpha x - \frac{ax^3}{3 \cos \alpha}$   
 $x (2g \sin \alpha - \frac{ax^2}{\cos \alpha}) = 0$   
 $x = 0 \vee 2g \sin \alpha = \frac{ax^2}{\cos \alpha}$   
 $x = \sqrt{\frac{2g \sin \alpha \cos \alpha}{a}} = \sqrt{\frac{2}{a} \sin \alpha \cos \alpha}$

Velocity at  $x = 0$ :  
 $\frac{dv}{dx} = 0$   
 $\frac{d}{dx} \left( \sqrt{2g \sin \alpha x - \frac{ax^3}{3 \cos \alpha}} \right) = 0$   
 $\frac{1}{2 \sqrt{2g \sin \alpha x - \frac{ax^3}{3 \cos \alpha}}} \cdot (2g \sin \alpha - \frac{ax^2}{\cos \alpha}) = 0$

Obratite pažnju na bitan deo zadatka da se prvi izvod brzine po vremenu može napisati kao proizvod prvog izvoda brzine po koordinati  $x$  i prvog izvoda koordinate  $x$  po vremenu.

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$$\begin{aligned}g(\sin \alpha - a \cos \alpha) &= 0 \\g \sin \alpha &= g a \cos \alpha \\x' &= \frac{\sin \alpha}{a \cos \alpha} = \frac{1}{a} \tan \alpha \\v_{\max} &= \sqrt{2g \left( \sin \alpha \frac{1}{a} \tan \alpha - \frac{\alpha}{2} \frac{1}{a^2} \tan^2 \alpha \cos \alpha \right)} = \\&= \sqrt{2g \left( \frac{1}{a} \frac{\sin^2 \alpha}{\cos \alpha} - \frac{1}{2a} \frac{\sin^2 \alpha}{\cos \alpha} \right)} = \\&= \sqrt{\frac{1}{2} g \frac{1}{a} \frac{\sin^2 \alpha}{\cos \alpha}} = \\&= \sqrt{\frac{g}{2a} \sin \alpha \tan \alpha}\end{aligned}$$

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### 6. Za samostalan rad (Domaći zadatak):

Automobil se kreće uzbrdo, po strmoj ravni nagibnog ugla  $\pi/3$ ,  
ravnomernom brzinom, pri čemu je vučna sila motora  $F_v = 9\text{kN}$ .

- a) Izračunati vučnu silu motora automobila ako bi se on kretao po horizontalnom putu gde je koeficijent trenja 2.5 puta manji od koeficijenta trenja između automobila i strme ravni. Pretpostaviti da se automobil kreće ravnomerno.
- b) Naći put koji telo pređe u prvom slučaju smatrajući da se automobil kreće celom dužinom strme ravni.

Masa automobile je 1t, a visina strme ravni 5m.